Lecture 20
Deformable / Non-Rigid Registration

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Registration: “Rigid” vs. Deformable

• Rigid Registration:
  ▪ Uses a simple transform, uniformly applied
  ▪ Rotations, translations, etc.
• Deformable Registration:
  ▪ Allows a non-uniform mapping between images
  ▪ Measure and/or correct small, varying discrepancies by deforming one image to match the other
  ▪ Usually only tractable for deformations of small spatial extent!

Deformable, i.e. Non-Rigid, Registration (NRR)

• Vector field (aka deformation field) T is computed from A to B
• Inverse warp transforms B into A’s coordinate system
• Not only do we get correspondences, but…
• We also get shape differences (from T)

NRR Clinical Background

• Internal organs are non-rigid
• The body can change posture
  ▪ Even skeletal arrangement can change
• Single-patient variations:
  ▪ Normal
  ▪ Pathological
  ▪ Treatment-related
• Inter-subject mapping: People are different!
  ▪ Atlas-based segmentation typically requires NRR

More Clinical Examples

• Physical brain deformation during neurosurgery
• Normal squishing, shifting and emptying of abdominal/pelvic organs and soft tissues
  ▪ Digestion, excretion, heart-beat, breathing, etc.
• Lung motion during respiration can be huge!
• Patient motion during image scanning

Optical Flow

• Traditionally for determining motion in video—assumes 2 sequential images
• Detects small shifts of small intensity patterns from one image to the next
• Output is a vector field, one vector for each small image patch/intensity pattern
• Basic gradient-based formulation assumes intensity values are conserved over time
**Optical Flow Assumptions**

- Images are a function of space and time
- After short time $dt$, the image has moved $dx$
- Velocity vector $v = dx/dt$ is the optical flow

$$I(x, t) = I(x + dx, t + dt) = I(x + v dt, t + dt)$$

- Resulting optical flow constraint:

$$C_v = I \cdot v + I_t = 0$$

**Optical Flow Constraint**

- Optical flow constraint dictates that when an image patch is spatially shifted over time, that it will retain its intensity values
- Let image $A = I(x, t=0)$ and let $B = I(x, t=1)$
- Then $I_t = A(T) - B$

This alone is not a sufficient constraint!

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**NRR Is Ill-Posed**

- Review of well-posed problems:
  - A solution exists, is unique, and depends continuously on the data
  - Otherwise, a problem is ill-posed
- Ambiguity within homogenous regions:

**Very Ill-Posed Problem**

- NRR answer is not unique, and...
- NRR Search-space is often $\infty$-dimensional!

- Solution: Regularization
  - Adding a regularization term can provide provable uniqueness and a computable subspace
- Usually base regularization on continuum mechanics
  - $T$ is restricted to be physically admissible
  - We're typically deforming physical anatomy, after all
- Optimum $T$ should deform “just enough” for alignment

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**NRR Regularization Methods**

- Numerous continuum mechanical models available for regularization priors
  - Elastic
  - Diffusion
  - Viscous
  - Flow
  - Curvature
- Optimization is then physical simulation over time, $t$, of trying to deform one image shape to match another
- This optimization has 3 equivalent formulations:
  - Global potential energy minimization
  - Variational or weak form, as used in finite-element methods
  - Euler-Lagrangian (E-L) equations, as used in finite-difference techniques

**Langrangian View**

- Elastic physical model:
  - How much have we stretched, etc., from our original image coordinates?
  - Simulation calculates the physical model's resistance to deformation based on the total deformation from time $t=0$ to now
- $T$ is the final vector field $\tilde{\delta}$:

$$\tilde{\delta} = \Delta x - \Delta \tilde{\delta}$$

$$A(X + \tilde{\delta}) - B(x)$$

$$X = x - \tilde{\delta}$$

- Deformation at time $t$:

$$\Delta X = X - X_0$$

- Deformation at time $t + dt$:

$$\Delta X = X - X_0$$
**Eulerian View**

- Viscous-flow physical model: How much have we flowed from our immediately previous simulation state?
- Simulation calculates the physical model’s resistance to deformation based on the incremental deformation from time \( t \) (now) to \( t + dx \).
- \( \dot{\mathbf{x}} \) is the aggregate flow of \( \mathbf{x}(t) \), based on accumulated optical flow (i.e., velocity) \( \mathbf{v}(t) \):
  - \( \dot{\mathbf{x}}(t) = \mathbf{x}(t) + \mathbf{v}(t) \)
  - \( \Delta \mathbf{x} = \mathbf{x}(t + dx) - \mathbf{x}(t) \)

**Deformation at time \( t \):**

**Deformation at time \( t + dx \):**

**Comparison of Regularization Reference Frames**

- Lagrangian
- The entire deformation is regularized
- Well constrained for “normal” physical deformation
- Too constrained to achieve “large” deformations
- Not ideal for many inter-subject mapping tasks

- Eulerian
- Only the incremental updates are regularized
- Underconstrained for “normal” physical deformation
- Readily achieves large, inter-subject deformations
- Unrealistic transformations can result

**Optical Flow Regularized**

\[
E_D(\mathbf{v}) = \int_\Omega \Psi(C_\mathbf{v}) d\Omega + \int_\Omega \Psi(\mathbf{v}) d\Omega
\]

- \( \Psi(\mathbf{v}) = \mathbf{v} \cdot \mathbf{v} \)
- \( \Psi(\mathbf{v}) = ||\mathbf{v}||^2 \)
- Goal: Minimize global potential energy, \( E_D \)
- First term adjusts \( \mathbf{v} \) to make the images match within the bounded domain \( \Omega \)
- Second term adds a stabilizing function \( \Psi \), typically a regulator operator \( L \) applied to \( \mathbf{v} \)

**Optical Flow E-L Regularized**

- After deriving the E-L equations & setting their derivative = 0, we find that the...
- Potential energy minimum will occur when:
  \[
  I_1 (L \cdot \mathbf{v} + I) - \mathbf{v} = 0
  \]
- First term minimizes optical flow constraint
- Second term minimizes Laplacian (i.e. roughness) of velocity field \( \mathbf{v} \)
- Note that this equation is evaluated locally
  - Allows for efficient implementation

**Demons Algorithm**

- Very efficient gradient-descent NRR algorithm
- Originally conceived as having “demons” push image level sets around, but is also...
- Based on E-L regularized optical flow
- Alternates between minimizing each half of the previous equation:
  - Descent in optical flow direction, based on:
    \[
    I_1 (L \cdot \mathbf{v} + I) = 0
    \]
  - Smoothing, which estimates \( \mathbf{v} = 0 \) with a difference-of-Gaussian filter, by applying a Gaussian on each iteration
### Demons Summarized

- Initialize solution (i.e. total vector field) = identity
- Loop:
  - Estimate vector field update
  - Use (stabilized) optical flow
  - Add update to total vector field
  - Blur total vector field (for regularization)

- Allows much larger deformation fields than optical flow alone.
- **Langrangian registration**: blur the total vector field (as above)
- **Eulerian registration**: blur the individual vector-field updates

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### Choices & Details

- There are many more NRR algorithms available
- Almost all of them are slower than demons, but they may give you better results
- See the text for details, and lots of helpful pictures